Inasmuch as vector multiplication is commutative if no term of the product is of a degree higher than the second in the vectors employed, and if the scalar part only of the resulting product be considered, we may assume that the algebraic identity (1) has a geometric interpretation which may be derived from (2) by considering the scalar part only of the vector products indicated in (2). The scalar part of the product of two vectors may be taken as the positive product of the lengths of the vectors into the cosine of their included angle. Placing this interpretation on the scalar part of the products indicated in (2) the equation of the problem is at once obtained, and the truth of the theorem established.

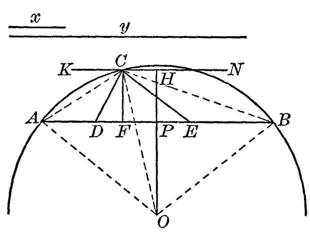
As no assumption has been made restricting any of the lines to any one plane the pentagon may, of course, be either plane or gauche. To help form a mental picture of a gauche pentagon, let $ABCD$ be a tetrahedron, with edges AB, BC, CD, DA, AC, and BD, and let E be a point within or without such tetrahedonr. Then if E be connected with A and D by right lines the figure $ABCDE$ will be a gauche pentagon.

445. Proposed by CLIFFORD N. MILLS, South Dakota State College.

Given the perimeter of a right triangle and the perpendicular falling from the right angle on the hypotenuse, to determine the sides of the triangle.

Solution by G. I. Hopkins, Manchester (N. H.) High School.

Let x be the altitude and y the perimeter. Draw $AB = y$, and OH its $\perp$ bisector. Make $PH = x$, and draw KN through $H \perp$ to HO. Make $PO = AP$. With O as center and radius OA describe the circle ACB. Draw the chords CA and CB, and the radii OA, OB, and OC. Make $\angle ACD = \angle CAD$, and $\angle BCE = \angle CBE$. $\therefore DCE$ is the $\triangle$ required. For,



$$\angle AOP = 45° = \angle POB. \quad \therefore \angle AOB = 90°.$$

$$\angle OAC = \angle OCA \quad \text{and} \quad \angle CAD = \angle ACD. \quad \therefore \angle OCD = \angle OAD = 45°.$$

In like manner $\angle OCE = 45°$; $\therefore \angle DCE = 90°$, $AD = DC$, and $EB = CE$; $\therefore$ the perimeter of the $\triangle DCE = y$, CF is $\perp$ to AB, $\therefore CF = HP = x$.

Note. The figure is not accurate as OP is not made equal to AP.

Also solved by B. J. Brown, A. H. Holmes, C. N. Schmall, and Nathan Altshiller.

CALCULUS.

356. Proposed by F. B. FINKEL, Drury College.

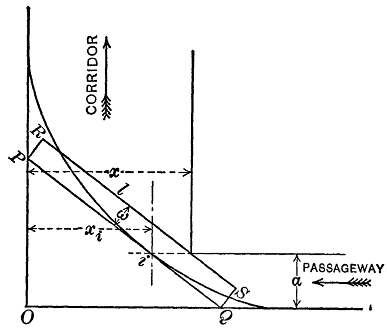
A steel girder l ft. long and w ft. wide is moved along a passageway a ft. wide and into a corridor at right angles to the passageway. How wide must the corridor be to admit the girder?

SOLUTION BY UNKNOWN AUTHOR, Stillwater, Okla.

Pass a horizontal plane through the passageway, corridor, and girder.

Let OP be the intersection with the corridor and OQ with the passageway. Take O as the origin of a system of rectangular coördinates. See the figure.

Suppose the ends P and Q of the girder to slip along the lines OP and OQ respectively.



As the end P is moved down the corridor the line PQ will move tangent to a curve such that the length of the tangent intercepted between the axes is the constant l.

The differential equation of such a curve is:

$$(1) \qquad y = px - \frac{lp}{\sqrt{1+p^2}}, \qquad \text{where} \qquad p = \frac{dy}{dx}.$$

Differentiating with respect to p, we have

$$p = p + x\frac{dp}{dx} - l\frac{dp}{dx} \cdot \frac{1}{(1+p^2)^{3/2}}.$$

Whence

$$(2) \qquad \frac{dp}{dx} = 0, \qquad \text{or } p = \text{constant},$$

which gives the general solution, and

$$(3) \qquad x - \frac{l}{(1+p^2)^{3/2}} = 0,$$

which gives the singular solution desired in this case.

Solving for p in (3) and substituting in (1) there results

$$y^{2/3} + x^{2/3} = l^{2/3}.$$

The intersection i of the line $y = a$ with this curve is at $x = (l^{2/3} - a^{2/3})^{3/2}$.

The equation of the line tangent at x_i, y_i is

$$y = x\frac{dy_i}{dx_i} + b, \qquad \frac{dy_i}{dx_i} = -\left(\frac{y_i}{x_i}\right)^{1/3} = -\frac{a^{1/3}}{[l^{2/3} - a^{2/3}]^{1/2}},$$

$$b = y_i - x_i\frac{dy_i}{dx_i} = a^{1/3}l^{2/3}.$$

The equation of the line RS parallel to PQ and distant ω from it is:

$$y = \frac{dy_i}{dx_i}x + a^{1/3}l^{2/3} + \omega\sqrt{1 + \left(\frac{dy_i}{dx_i}\right)^2}.$$

Substituting and simplifying,

$$y = -\frac{a^{1/3}x}{[l^{2/3} - a^{2/3}]^{1/2}} + a^{1/3}l^{2/3} + \frac{wl^{1/3}}{[l^{2/3} - a^{2/3}]^{1/2}}.$$

The intersection of this line with the line $y = a$ is at $x = (l^{2/3} - a^{2/3})^{3/2} + w(l^{1/3}/a^{1/3})$, which is the required width.

Also solved by W. A. FLANAGAN and L. SIVIAN.

357. Proposed by W. D. CAIRNS, Oberlin College.

A continuous variable represented by a point on a vertical line changes according to such a law that it is reduced to $1/m$ of its value on being moved a units upward, irrespective of the special position from which it is moved. Find the law of change, that is, the relation between the variable y and the height h of the variable point above a fixed point of the vertical line.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We are to solve the functional equation $f(h + a) = (1/m)f(h)$. Taking logarithms of both sides we have the equation, $\log f(h + a) = \log f(h) - \log m$. Let $\psi(h) = \log f(h) + (h/a) \log m$. Then $\log f(h + a) = \psi(h + a) - [(h + a/a] \log m$, and $\log f(h) = \psi(h) - (h/a) \log (m)$. The above logarithmic equation becomes $\psi(h + a) = \psi(h)$, which is satisfied by any periodic function of period a. Then the function $f(h) = e^{\psi(h)-(h/a)\log m} = m^{-(h/a)} \cdot e^{\psi(h)}$ or $f(h) = m^{-(h/a)} \cdot P(h)$ where $P(h)$ is any periodic function of period a.

Also solved by JOSEPH NYBERG, J. W. CLAWSON, and the PROPOSER.

MECHANICS.

289. Proposed by C. N. SCHMALL, New York City.

A particle of elasticity e is projected with a velocity v at an angle of elevation φ from a point on a smooth horizontal plane. Show that after $\dfrac{2v \sin \varphi}{g(1 - e)}$ seconds it will cease to rebound and will move along the plane with a uniform velocity $v \cos \varphi$.

SOLUTION BY A. M. HARDING, University of Arkansas.

Let t_1 = time from A to A_1, t_2 = time from A_1 to A_2, etc. Equating the normal components before and after each impact we obtain

$$\begin{gathered} v_1 \sin \varphi_1 = ev \sin \varphi, \qquad v_2 \sin \varphi_2 = ev_1 \sin \varphi_1 = e^2v \sin \varphi, \\ v_3 \sin \varphi_3 = ev_2 \sin \varphi_2 = e^3v \sin \varphi, \text{ etc.} \end{gathered} \tag{1}$$